30th Annual International IEEE EMBS Conference
Vancouver, British Columbia, Canada, August 20-24, 2008

# Simulation of Organ Deformation using Boundary Element Method and Meshless Shape Matching

Bo Zhu , Lixu Gu*, Jingsi Zhang, Zhennan Yan, Lei Pan and Qiang Zhao*

*Abstract*—**In this paper, a novel approach to perform real-time simulation of the deformation of anatomical organs is proposed for virtual surgery study. The method, which is both physics and interactivity motivated, is composed of two algorithms: boundary element method and meshless shape matching. We employ boundary element method to simulate a precise global deformation, and use meshless shape matching method to achieve low latency. In addition, a state machine is applied to control the computation patterns of deformation. The initial experiment reveals that the proposed approach can simulate organ deformations both efficiently and accurately.**

## I. Introduction

THE rising complexity and costs of surgical training and development of new surgical procedures make virtual surgery simulations increasingly important. One of the essential parts of such a system is simulating deformable organs to response to interventions using surgical instruments. Accurate and robust deformable anatomical models could significantly improve the realistic behavior of surgical training systems.

Despite the long history of deformable modeling in the field of virtual reality, research results cannot be easily applied in virtual surgery. It is difficult to simulate soft biological tissues in real time because of the conflicting demands of interactivity and accuracy. In surgical training environments, both the deformation of the organs and the feedback forces must be computed in real time. On the other hand, the shape changes of the soft tissues and the contact forces felt by the surgeon should be accurate in order to let the surgeon acquire useful manual skills for real surgeries, rather than skills at a video game. Hence, a suitable model for soft tissues in virtual surgery requires both computational efficiency and physical accuracy. Unfortunately, many available techniques for simulating deformable objects fail with respect to one of the two aspects.

According to the two aspects, the existing deformable models can be classified into two categories – interactive models and physically-based models. Interactive models are widely used in computer games. In these models, speed and low latency are paramount. Typical examples include meshless shape matching models [5], modal analysis models [10] and mass-spring models. Physically-based models play an important part in computer animation, which are intended to simulate the dynamic object behavior as realistically as possible. Typical contributions to this research area are boundary element method proposed by James and Pai [1-3] and finite element method [6]. However, these models are computationally expensive and cannot simulate objects with high resolution.

To overcome the existing restrictions of the two kinds of models, in this paper we propose a new method that balance the priority of efficiency and accuracy in simulating deformations of soft tissues. Central to our approach are two deformation algorithms – boundary element method and meshless shape matching. Boundary element method is a physically-based approach, which is used in computing global deformation basis when an external force is added on the object. Meshless shape matching is a fast interactive approach and we use it to simulate the vibration behavior of the soft tissue. In our method, the two algorithms is combined to simulate one deformable object by using a state machine model to analyze the force condition on the object and decide which deformation computation pattern to use. Since our method takes both accuracy and interactivity into consideration, it is a compromise deformation modeling method and is especially suitable for simulating deformable anatomical organs in virtual surgery.

## II. Algorithm Overview

In our approach, the key to balance accuracy and interactivity is a state machine which clearly and simply handles the switch between deformation algorithms. This state machine includes three main states – deforming, restoring and rest.

When an external force is detected on the object, the state will changed to 'deforming'. In this transition, new global deformation will be computed in response to the external force. To simulate the deformation realistically, the boundary element method is used to compute the global deformation basis based on Navier's Equation. While standard BEM approaches need to update deformation basis in every time step, this process is only required when state has changed to 'deforming' in our case, which will greatly reduce computation complexity.

When an external force is removed from the surface, the state moves to 'restoring'. In this state, the deformed surface needs to restore to its original shape, and the meshless shape matching algorithm provides an efficient way to simulate this process. The basic idea behind this algorithm is simple: the object is simulated as a simple particle system without particle-particle interaction. After each time step, each element is pulled towards its goal position.

Bo Zhu, Lixu Gu, Jingsi Zhang, Zhennan Yan and Lei Pan are with the Laboratory of Image Guided Surgery and Therapy (IGST), Shanghai Jiao Tong University (SJTU), China. Qiang Zhao is with Shanghai Zhongshan hospital, Shanghai, China
*Equal corresponding authors.

978-1-4244-1815-2/08/$25.00 ©2008 IEEE. 

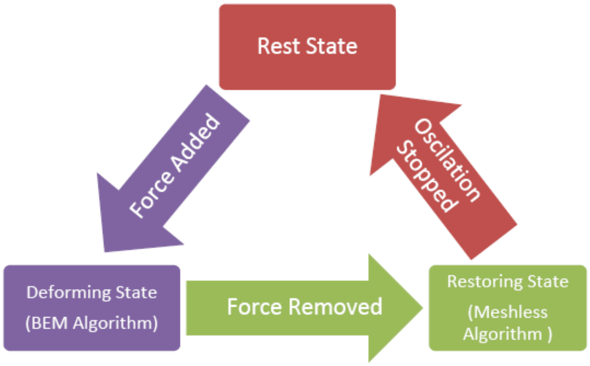

Fig 1: State machine controlling the computation patterns

## III. Boundary Element Method

### 3.1 Physical Principles

Our model is physically based, the deformation of which is governed by the well-known Navier's equation of linear elastostatic objects. It is written as

$$\mathbf{Nu} + \mathbf{b} = 0 \tag{1}$$

in which **N** is a linear second-order differential operator, **u** is the displacements of points in the domain, and **b** is a term of body force such as gravity. We denote the domain of the deformable object by $\Omega$ and denote the boundary by $\Gamma$. The change in shape of the surface is described by the displacement filed $u(x), x \in \Gamma_u$, and the force distribution is the traction filed $p(x)$, $x \in \Gamma_p$. According to the Navier's equation, for $x \in \Omega$, displacements $u_k$ and tractions $p_k$ satisfy the following equations.

$$\begin{cases} \lambda u_{k,ki} + G\left(u_{i,\ jj} + u_{j,\ ij}\right) + f_i = 0 & x \in \Omega \\ u_i = \bar{u}_i & x \in \Gamma_u \\ p_i = \sigma_{ij} n_i = \bar{p}_i & x \in \Gamma_p \end{cases} \tag{2}$$

Where i, j and k range from 1 to 3, representing the three dimensions in $\mathbb{R}^3$ .Using Green-Gauss Theorem and the Kelvin fundamental solutions of linear elastostatic problem, equation on a domain could be converted to an integral equation defined on the surface of that domain.

$$C_{lk}(P')u_k(P') = \int \Gamma \left[u_{lk}^*(P',Q') - u_k(Q')p_{lk}^*(P',Q')\right] d\Gamma(Q') + \int \Omega\, u_{lk}^*(P',Q) f_k(Q) d\Omega(Q) \tag{3}$$

Where $u_{lk}^*(P',Q')$ and $p_{lk}^*(P',Q')$ are fundamental solutions of Navier's equation. Much of the material about the fundamental solutions is presented in greater detail in [6].

### 3.2 Surface Subdivision

To solve the boundary integral formulation (3) in a numerical way, the surface of the organ model is divided into a set of non-overlapping elements. The surface subdivision of BEM is compatible with geometric models used in VTK, OpenGL or DirectX. As the anatomical organ models rendered in these graphics interfaces are mostly surface models, the BEM model can utilize them without extra data structures. After dividing surface into n elements, we assume that the surface field is parametrized by n nodal variables. For each node x, it is assumed that either displacement field u(x) or traction field p(x) is specified.

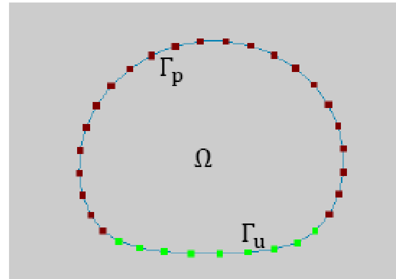

Fig 2: The boundary of the domain $\Omega$ can be divided into $\Gamma_p$ ( traction specified ) and $\Gamma_u$ ( displacement specified ).

Substituting n boundary elements into the boundary integral equation, we would get the dicretization form of the elasticity integral equation applied at $P_i$:

$$Cu_i + \sum_{j=1}^{n} \widehat{H}_{ij} u_j = \sum_{j=1}^{n} G_{ij} p_j \tag{4}$$

Let

$$H_{ij} = \widehat{H}_{ij} + C\delta_{ij} \tag{5}$$

Assembling the N ( N = 3n ) equations at all surface elements into a block matrix system yields

$$HU = GP \tag{6}$$

According to the different boundary value type ( p or u ) of each element, we exchange the corresponding block columns of H and G, and move all the unknowns to the left-hand side to get the final system of linear algebraic equations:

$$AX = C\overline{X} \tag{7}$$

This system of N linear algebraic equations contains N unknown deformations and tractions in vector X, so it is possible to solve the equations to determine deformation displacements and response forces.

### 3.3 Deformation Basis

In classical BEM approach, it is very computational expensive to solve the system of boundary equations in every time step. In our approach, we use an acceleration technique called deformation basis to simulate deformation in interactive rate. The key point to this technique is to pre-compute the deformation-basis for each element in run time. With the deformation basis, deformations in response to different external forces could be computed in real-time by accordingly scaling the basis.

To solve the deformation basis in real-time, $A^{-1}$, the inverse matrix of A could be pre-computed, and therefore

$$X = A^{-1}C\overline{X} \tag{8}$$

After this pre-computing process, the entire solution could be obtained only by computing a matrix-vector product using the non-zero elements of the vector $\overline{X}$.

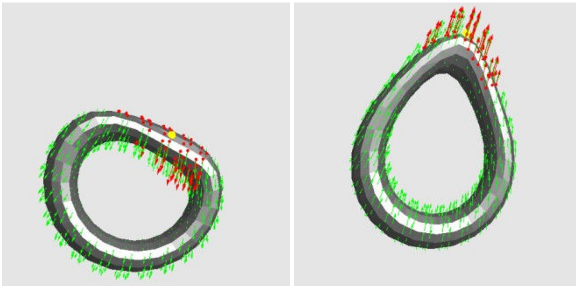

Fig 3: Deformation basis ( green arrows ) for each surface element are computed in real-time to control the global deformation in response to external forces( red arrows ).

### 3.4 Material Property

In order to enhance the reality in virtual surgery, it is important to simulate the different material properties of different anatomical organs and soft tissues. For example, some tissues are very sponge, while some others are very dense.

In the BEM model, it is possible to specify the deformation behaviors of different materials using a few material properties, rather than adjusting a huge number of spring parameters in mass-spring models or center-line models ( see experiment 1 and 2 ). To simulate the response of soft tissues, only two independent parameters, shear modulus and Poisson's ratio, are needed in our case. Shear modulus is a measure of the stiffness of a given material, and Poisson's ratio is a measure of the tendency of a material to get thinner in other two directions when being stretched in one direction. It is easy to measure these parameters of vivo or ex vivo tissues by using some devices such as TeMPest[8].

## IV. Meshless Shape Matching

When an external force is removed from the object, it is necessary to simulate the process that the object restores to its original shape. Considering that there is no new deformation basis generated in this process, we only need to set up a correspondence between the deformed positions and original positions. After each time step, each element is pulled toward its goal position. In contrast to the aforementioned physically-based algorithm, the object is simulated as a particle system without particle-particle interaction, and no connectivity information or mesh is needed in this approach. Therefore its efficiency in terms of memory and computational complexity is very high.

Our approach draws from the previous work in the field of shape matching [5, 7]. While most of the shape matching approaches establish the correspondences between two shape representations by using rotation and translation matrix[5], in our case we simply use a set of springs connecting two groups of points. For each point pair in the matching correspondence, the undeformed point is located at the origin p( 0 ) while the deformed point is free and located at p( x ), the free point is pulled towards the equilibrium p( x ) = p( 0 ) by the spring force

$$\mathbf{f} = -\mathbf{k}[\, \mathrm{p}(\mathbf{x}) - \mathrm{p}(\mathbf{0}) \,] - \mathbf{f}_{\mathrm{damp}} \qquad (9)$$

Due to the existence of damping force, the deformed points will finally stay at the original locations, and the drastic degree and fastness of this deformation process could be controlled by simply change the stiffness coefficient **k** and damping force $f_{damp}$ .

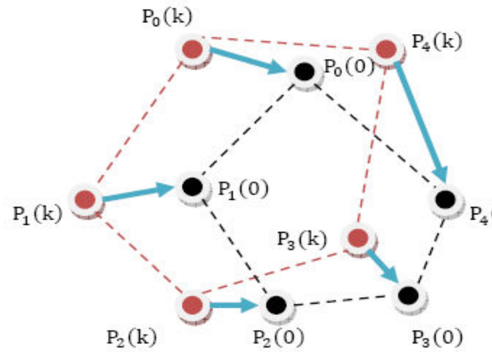


Fig 6: Point pairs in the matching correspondence are connected by a set of springs ( blue arrows ), and the deformed points( red ) are pulled toward their original shapes( black ).

## V. Experiments

We have integrated our method in a simulation environment for deformable anatomical organs. Several experiments have been carried out to test the quality and performance of the proposed method. All test scenarios presented in this section have been performed on a PC Pentium 4, 1.7GHz, GeForce 6800GPU.

In a first test, we test the compression behavior of deformable object with Poisson ratio $\nu = 0.45$. The coarse reference mesh is shown as the original shape to illustrate the shrink( fig right ) and bulge( fig middle ) behavior of the deformed object.

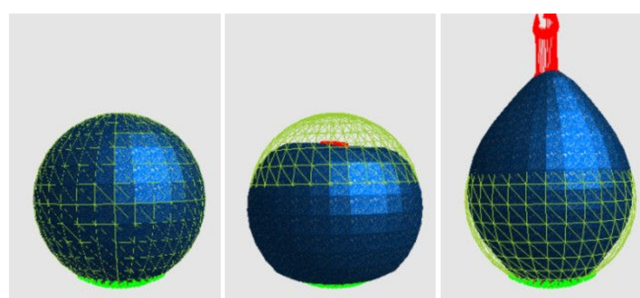

Fig 4: Compressibility of objects with Poisson ratio $\nu = 0.45$

In the second scenario, the relation between values of shear modulus and rigidness of objects is verified by adding the same external forces ( pull in the first row and push in the second row ) to kidney models with the same geometry structure but different values of shear modulus. The two rows of pictures illustrate the different deformation shapes.

The update time of computing global deformation shapes in response to external forces by solving linear equations ( in the standard BEM ) and by computing deformation basis ( in our approach ) is compared in the third experiment. As shown in table 1, the update time of our approach scales linearly with the number of model polygons.

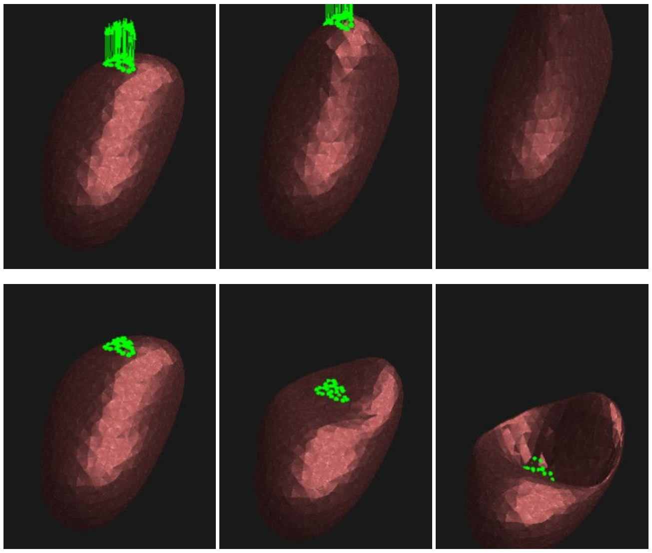


Fig 5: Deformation of kidney with different shear modulus

Table 1: Update time of standard BEM and deformation basis

| Model | Number of vertexes | Number of polygons | Update using standard BEM(ms) | Update using deformation basis(ms) |
|---|---|---|---|---|
| Tissue | 230 | 456 | 92 | 16 |
| Tumor | 482 | 960 | 251 | 31 |
| Kidney | 1294 | 2584 | 703 | 125 |

## VI. CONCLUSIONS AND FUTURE WORK

In this paper we have presented a new approach to simulate soft biological deformations. The underlying model is related to both physics motivated approach and interaction motivated approach. Our model is efficient to compute, and provides unconditionally stable simulations in processing interactions between rigid surgical instruments and soft tissues. This model is a part of our laparoscope-based surgery training system. We have investigated interaction between deformable objects and fluids as one of the extensions, ongoing work focuses on further extensions such as blood simulation.

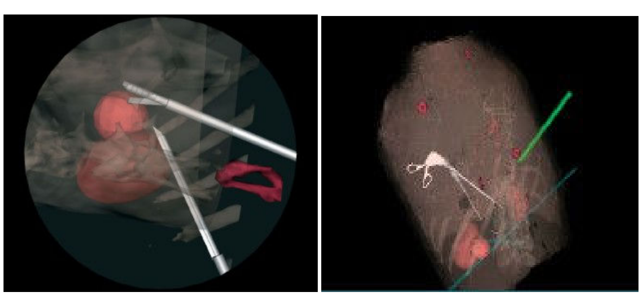

Fig 7: The laparoscope-based surgery training system


## ACKNOWLEDGEMENTS

This paper is partially supported by the Chinese National 863 research foundation under Grand No. 2006AA02Z310 and 2007AA01Z312, the Chinese National Natural Science Foundation under Grant No. 60571061.We thank to all the members in the laboratory of Image-Guided Surgery and Therapy of SJTU for supporting the groundwork of this research.


## REFERENCES


[1] Jung Kim, Changmok Choi, Suvranu De and M. A.. Srinivasan. Virtual surgery simulation for medical training using multi-resolution organ models. Int J Med Robotics Comput Assist Surg, 3: 149–158, 2007;.

[2] James DL, Pai DK. Multi-resolution green's function methods for interactive simulation of large-scale elastostatic objects. In ACM Transactions on Graphics. Association for Computing Machinery: ACM Press, NY, USA; 47–82, 2003.

[3] James, D. L., And Pai, D. K.. Artdefo, accurate real time deformable objects. In Computer Graphics Proceedings, Annual Conference Series, ACM SIGGRAPH 99, 65–72, 1999.

[4] Chanthasopeephan T, Desai JP, Lau AC. Modeling soft-tissue deformation prior to cutting for surgical simulation: finite element analysis and study of cutting parameters, IEEE Trans Biomed Eng; 54(3): 349–359, 2007.

[5] Muller, M., Heidelberger, B., Teschner, M., and Gross, M. Meshless deformations based on shape matching. ACM Transactions on Graphics, 24-3, 471-478, 2005.

[6] C.A.Brebbia, J.C.F.Telles and L.C. Wrobel. Boundary Element Techniques: Theory and Applications in Engineering, Springer-Verlag, New York, 1984.

[7] Alexa, M., Cohen-OR, D., and Levin, D., As-rigid as-possible shape interpolation. In Computer Graphics Proceedings, Annual Conference Series, ACM SIGGRAPH 2000, 157–164, 2000.

[8] Ottensmeyer MP, Salisbury JK., In vivo data acquisition instrument for solid organ mechanical property measurement. Proc. of Medical Image Computing and Computer-Assisted Intervention Conference (MICCAI 2001), Utrecht, Netherlands, 975–982, 2001.

[9] Seymour NE, Rotnes JS. Challenges to the development of complex virtual reality surgical simulations. Surg Endosc 2006; 20(11): 1774–1777.

[10] Hauser, K. K., Shen, C., and O'Brien, J. F., Interactive deformation using modal analysis with constraints. In Graphics Interface, A. K. Peters, CIPS, Canadian Human Computer Communication Society, 247–256,2003.

[11] Tendick F, Downes M, Goktekin T, *et al.* A virtual environment testbed for training laparoscopic surgical skills. Presence Tele-operators Virtual Environ 2000; 9(3):236–255.